

**AM120 program - Mathematical Analysis 2**

(CdL Mat, 9 cfu, code: 20410388)

**Part 1: Axiomatics of  $\mathbf{R}$  and its main subsets Axiomatic definition of  $\mathbf{R}$ .**

Inductive assemblies; definition of  $\mathbf{N}$  and induction principle. Definition of  $\mathbf{Z}$  and  $\mathbf{Q}$ ;  $\mathbf{Z}$  is a ring,  $\mathbf{Q}$  is a field.

$n$ th roots; rational powers.

**Part 2: Theory of Limits**

The extended line  $\mathbf{R}^*$ : intervals, neighbourhoods and accumulation points.

Limits of functions in  $\mathbf{R}^*$ .

Comparison theorems.

Lateral limits; limits of monotone functions.

Algebra of limits on  $\mathbf{R}$  and  $\mathbf{R}^*$ .

Composition limit of functions.

Limits of inverse functions.

Notable limits. The number of Napier. Exponential and trigonometric functions.

**Part 3: Continuous functions**

Topology of  $\mathbf{R}$ .

Theorem of existence of zeroes. Bolzano-Weierstrass theorems. Weierstrass' Theorem.

Uniformly continuous functions.

**Part 4: Differentiable functions**

Rules of derivation. Derivatives of elementary functions.

Local minima and maxima and elementary theorems on derivatives (Fermat, Rolle, Cauchy, Lagrange).

Bernoulli-Hopital theorem.

Convexity.

Taylor's formulae.

**Part 5: Riemann integral in  $\mathbf{R}$ .**

The Riemann integral and its fundamental properties.

Integration criteria. Integrability of continuous and monotone functions.

The fundamental theorem of calculus and its applications (integration by parts, changes of variables in integration). Generalized ("improper") integrals and related integrability criteria.